

ELECTRONIC FUNDAMENTALS

THEORY LESSON 14

INDUCTANCE

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RCA INSTITUTES, INC.
A SERVICE OF RADIO CORPORATION OF AMERICA
HOME STUDY SCHOOL
350 West 4th Street, New York 14, N. Y.

Theory Lesson 14

INTRODUCTION

Earlier in this course you were told that you needed to know about three circuit properties in order to understand radio theory. You have already learned about the first of these — resistance. To prepare for your study of the second of these properties — *inductance* — you learned something about alternating current. You did this because inductance affects a-c circuits much more than it does d-c circuits. In this lesson you will learn about inductance. Don't try to rush through this lesson — take it easy. Think about each new idea as you read, and make sure that you understand it before you go on to the next idea. This lesson is so very important to your later understanding of radio and television theory that it will pay you to take the time to study it carefully.

14-1. SELF-INDUCTION IN D-C CIRCUITS

Inductance is the property of a circuit that opposes a change in amount of current. Any circuit component that is designed to introduce inductance into a circuit is called an *inductor*. In d-c circuits, the current is always in the same direction and always has the same value. It changes only when the circuit is being closed or opened. In a-c circuits, however, the current changes direction twice in every cycle, and may change in value throughout the entire cycle. This makes inductance a more important property of a-c circuits than of d-c circuits. In order to get a better understanding of inductance, we will first consider a d-c circuit because you are more familiar with d-c circuits, and because direct current stays the same long enough for you to follow what happens more easily.

Let us begin with the simple circuit shown in Fig. 14-1a. It contains a battery,

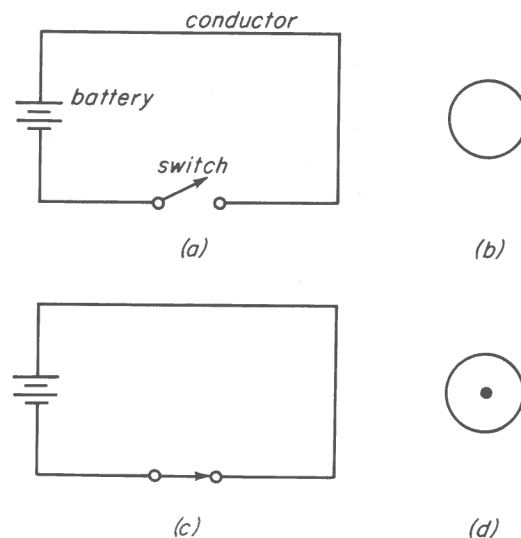


Fig. 14-1

an open switch, and a conductor. Because the circuit is open, there is no current flow — as the cross-section view of the conductor (Fig. 14-1b) shows.

As the switch is closed and the battery voltage is applied to the circuit (Fig. 14-1c), many free electrons in all parts of the conductor instantly start passing from one atom to another to form a current of electrons in the direction shown in Fig. 14-1d. (You remember that a dot in the center of the cross-section means the current is coming toward us.) The cross-sectional view, Fig. 14-2a, shows some of these moving free electrons. Because they are moving, each electron is surrounded by a magnetic field. In Fig. 14-2b, one such electron is represented surrounded by its magnetic field. However, each and every electron in motion has such a field. Because the direction of motion is the same, the direction of each field is the same. As you saw in the lesson on electromagnetism, magnetic flux lines tend to aid each other; so the field of one electron adds to the field of the next electron. Figure 14-2c shows some of these combined flux

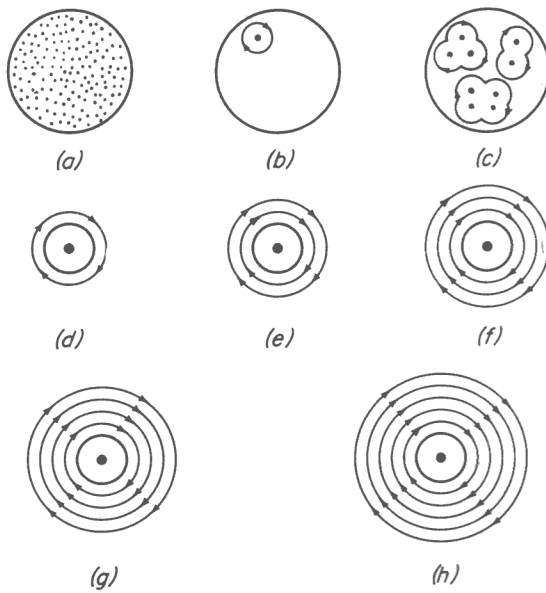


Fig. 14-2

lines. As a result of *all* of the flux lines of *all* of the moving electrons combining, one large flux line is formed that completely surrounds all of the moving electrons and moves outside the conductor itself, as shown in Fig. 14-2d. At this instant there is only one clockwise flux line surrounding the conductor. As the current continues to increase in the conductor, more flux lines are formed and move out from the conductor. Figure 14-2e shows that the first line has grown larger and has moved farther out from the center of the conductor and that a second line has formed closer to the wire. Its direction is also clockwise, as shown in the figure. In Fig. 14-2f and g, the number of lines is still increasing. Finally, in Fig. 14-2h, the current has reached its greatest value, and the magnetic field now has its greatest size. Actually, the field may now have thousands of flux lines, but to make the drawing simpler, only five are shown. Remember that it took a few instants for the current to grow to its largest value. While the current was growing, the magnetic field also grew. When both the current and the magnetic field are maximum, they remain so until the switch is opened again.

Before we learn what happens to the field when the switch is opened, let us see what happened while the field of Fig. 14-2h was

being formed. You remember from your study of electric generators and transformers in Lesson 12 that a voltage is induced in a conductor that is moving within a magnetic field. As you know, this is only *one* way in which a voltage may be induced. Another way is to have the magnetic field moving or building up around the conductor. This is exactly what was happening in Fig. 14-2. While the magnetic field was forming, the lines of force were moving out from the cross section of the conductor. Therefore, a voltage must have been induced in the conductor *itself*. Let us see how this induced voltage acts. Remember the left-hand rule you learned for induced voltage in Lesson 12. It says that if the thumb, forefinger, and middle finger of the left hand are held at right angles to each other, with the thumb pointing in the direction of motion of the conductor with respect to the magnetic field and the forefinger pointing in the direction of the lines of force, the middle finger will point in the direction of the induced voltage. Actually your middle finger points in the direction that an induced current would tend to flow, and we can use this information to determine the polarity of the induced voltage.

Now let us apply this rule to the case shown in Fig. 14-2. First we will choose a convenient point on the outside of the cross section of the conductor that we have been talking about. We still have to make one thing clear before we can apply the rule. Figure 14-3a shows the cross section of the conductor with a point marked *A* at the left. Point *A* represents any point on the circumference of the cross section of the conductor; what is true of the action at point *A* is true of the action at any point. The flux lines that formed as the switch closed moved out in a direction away from the center of the cross section of the conductor, as shown by the arrow. If the flux moves in this direction when it passes through point *A*, the effect is the same as if the magnetic field stood still and the conductor were moving in the opposite direction, as shown in Fig. 14-3b. So the

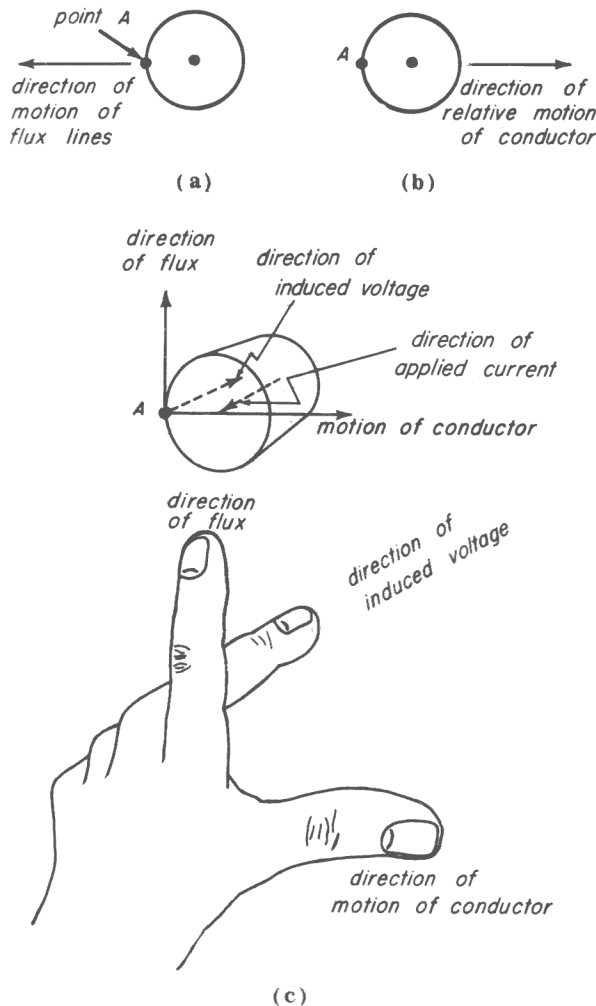


Fig. 14-3

relative motion of the conductor is to the right, which we must remember when we apply the left-hand rule in Fig. 14-3c. The direction of the magnetic field at point A is up. Now try to place your left hand in the position shown in Fig. 14-3c. The thumb is pointing to the right, in the direction in which the conductor appears to be moving, the forefinger points up in the direction of the field, and the middle finger points into the center of the cross section. But the original current from the battery was flowing out of the cross section. This means that the direction an induced current would tend to flow is opposite to that of the current from the battery. Therefore, the induced voltage must be opposing the battery voltage.

Since the magnetic lines of force move only while the current is building up, a voltage is induced in the conductor only during that time. As the current builds up to

its maximum, the induced voltage becomes smaller and smaller. Finally, when the current is maximum and the magnetic field is no longer changing in size, the induced voltage falls to zero. Because the voltage induced in the conductor is due to its own changing current, we call the process *self-induction*. Also, because the induced voltage opposes the applied voltage from the battery, we call it a *counter-emf*, meaning opposing voltage.

Once the current has reached its steady maximum value, and the induced voltage has dropped to zero, the induced voltage will remain zero as long as the switch remains closed. Now let us see what happens when the switch is suddenly opened. With only resistance in the circuit, the current would immediately fall to zero. But we are considering a conductor around which a magnetic field has been set up. As the switch is opened, the magnetic field starts to collapse. This means that the lines of force start to shrink back toward the center of the conductor. Look at Fig. 14-4a, which shows the same cross section as Fig. 14-3. Since the field is shrinking toward point A, or to the right, you can see that the effective motion of the conductor at point A now appears to be to the left. The direction of the flux lines is still up. So, if you place your left hand with the thumb pointing to the left and the forefinger pointing up, the middle finger will now point out of the center of the cross section. Once again a voltage has been induced in the conductor, but this time the induced voltage has the same polarity as the battery voltage. This means that the induced voltage tends to keep the original current flowing for an instant after the switch is opened, or keeps it from falling to zero immediately. Therefore, as the current falls towards zero after the switch is opened, the magnetic field around the conductor collapses. The magnetic lines of force cut the conductor, inducing a voltage that opposes the change in conductor current as it falls to zero. The induced voltage will keep current flowing until the energy stored in the magnetic field is used up and no magnetism remains.

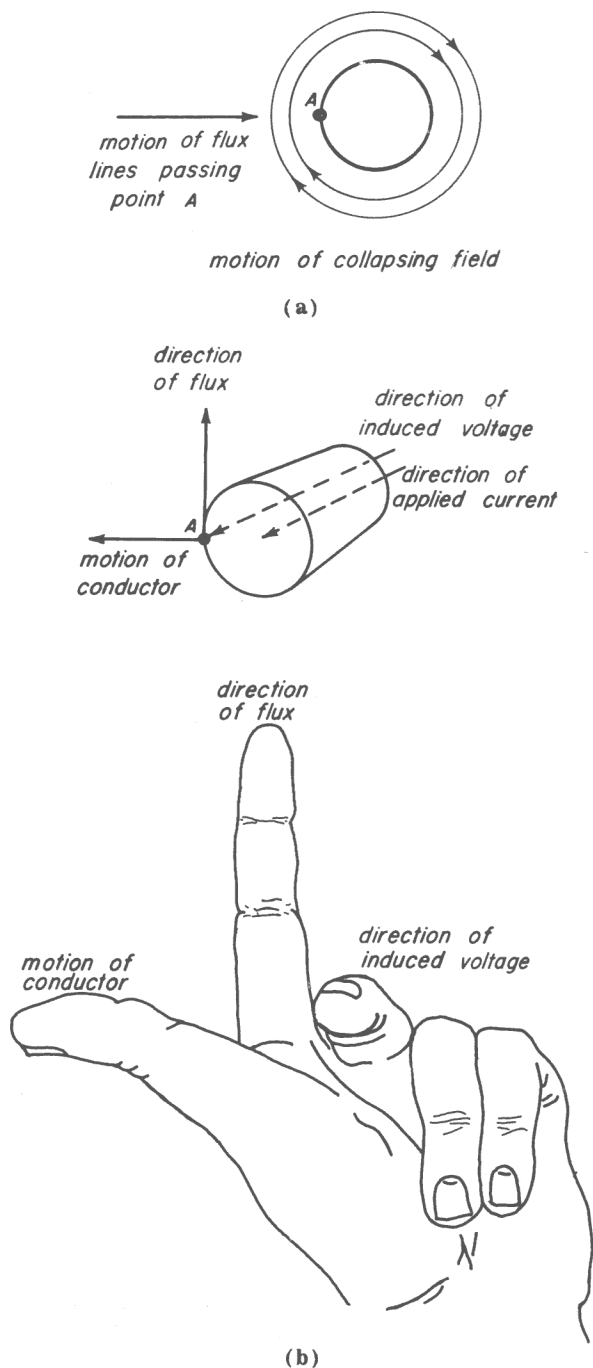


Fig. 14-4

The induced voltage appears only when the conductor current changes — when the switch is closed and again when it is opened. When the switch is closed, the expanding magnetic field cuts the conductor, inducing a voltage whose polarity *opposes* the battery voltage (Fig. 14-3c). When the switch is opened, the induced voltage has *same* polarity as the battery voltage (Fig. 14-4b). The polarity of the induced voltage always opposes the change in current. This property of induced voltages in inductors

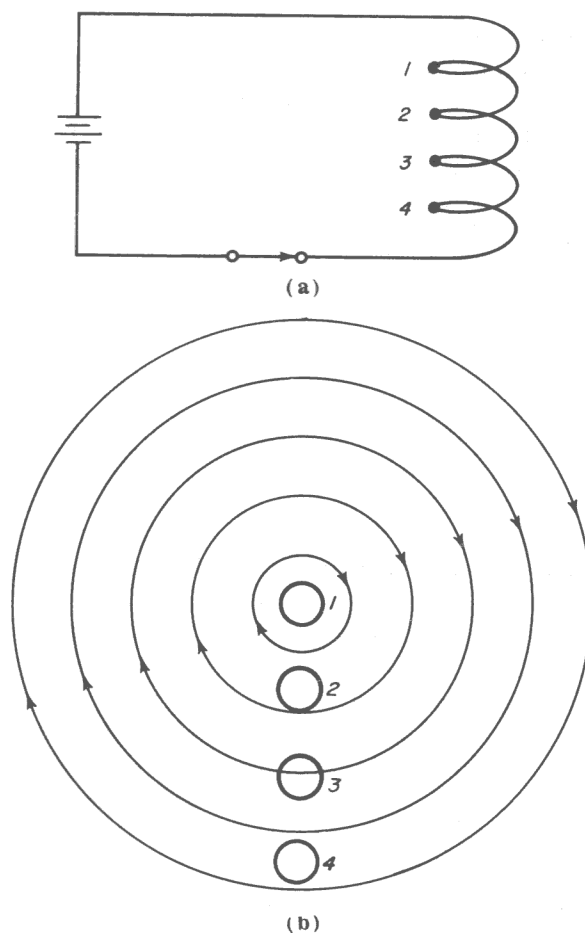


Fig. 14-5

was first observed by a German scientist named Lenz who said that the induced voltage opposes the change in current producing it. We call this Lenz's law. Since the induced voltage always opposes, it is called the counter emf (abbreviated *cemf*). The property of a conductor or circuit to produce such a counter emf is called self-inductance.

Self-Inductance in a Coil. If we wind our conductor into a coil of several turns, we find that the effect of self inductance increases. Let us see why this should be so. Figure 14-5a shows a coil connected to a battery through a switch. Figure 14-5b shows a cross-section view of each turn of the coil and the field that has formed around the first turn of the coil. (In looking at this drawing, bear in mind that a similar field is formed around each conductor.) Notice that the flux lines that move out from the first turn also cut the other turns, which means that an emf is induced not only in the first turn, but in other turns as well. This is

equally true when the switch is opened and the field collapses. As a result, the total cemf induced in the coil when the field is forming and when it is collapsing is greater than it would be if the coil were unwound and connected to the battery as a single conductor. The amount of cemf produced in a coil or inductor depends on two factors

1. The more turns or the greater the inductance, the greater is the cemf.

2. The faster the current changes (increases or decreases), the more cemf is produced in a given coil.

14-2. MEASUREMENTS OF INDUCTANCE

Now we are ready to get a clearer meaning of the term *inductance*. We have already defined inductance as the property of a coil that opposes a change in current. We have seen how this was true by tracing the action in a circuit with self-inductance. Here we found that an induced voltage was produced only when the current started or stopped flowing. Induced voltages can also be produced when the amount of current flowing in a circuit with self-inductance is changed. For example, suppose we substituted a rheostat for the switch in Fig. 14-5a. We could then change the amount of current flow through the coil. The faster we turned the rheostat, the faster the current would change, and the higher the induced voltage would be.

So we can say that the size of the voltage induced in a coil depends on the rate of change of current flowing through it.

When you were learning about resistance, we defined resistance as a property of a conductor that opposes the flow of current and we also gave it *numerical* (number) values. For example, we can say that a certain wire has a resistance and that the amount of resistance is 100 ohms. The same thing is true about the term inductance. We use inductance to describe a property of a coil, and we can measure or express the amount of inductance that a coil has. We use the unit *henry* to express the value of inductance of a coil. We can define the

unit henry by using the relationship between induced voltage and current change in a coil. A coil has an inductance of one *henry* when a voltage of *one volt* is induced in it by a current which is changing at the rate of *one ampere per second*. We can use the following formula to find the inductance of a coil if we know the value of voltage induced in it and the rate of change of the current flowing through it:

$$\text{Inductance (in henrys)} = \frac{\text{Voltage induced in coil (volt)}}{\text{Current change in coil (amp/sec)}}$$

For example, if a coil is found to have an induced voltage of 50 volts at an instant when the current is changing at the rate of 25 amperes per second, the inductance of the coil must be 50/25, or 2 henrys.

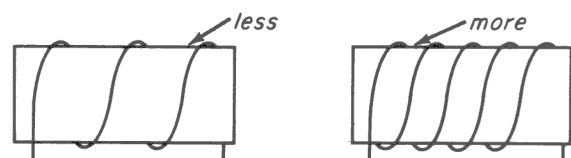
For most radio work, one henry of inductance is actually much greater than the inductance you will have to use in a circuit. For this reason, servicemen usually use the unit *millihenry* (1 millihenry = 0.001 henry), or *microhenry* (1 microhenry = 0.000001 henry). The abbreviation used for millihenry is *mh*, and that used for microhenry is μh . To give you an idea of the size of coils used in radio work, the antenna coil of a broadcast receiver may have an inductance of 250 μh , while the coil in an i-f stage of a television may have an inductance of only 5 μh .

The inductance of a coil depends on certain physical factors. They include:

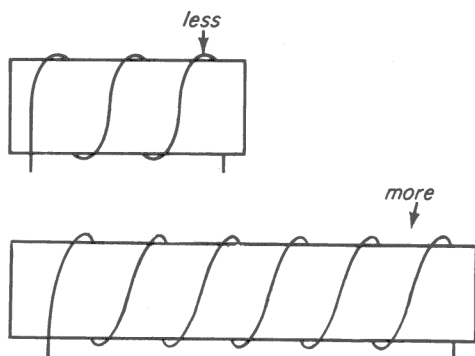
1. *The number of turns per inch of coil lengths.* If two coils have the same length, as in Fig. 14-6a, the one with the greater number of turns has the greater inductance.

2. *The length of the coil.* If two coils have the same number of turns per inch of length and one of the coils is longer than the other, the longer coil has the greater inductance (Fig. 14-6b).

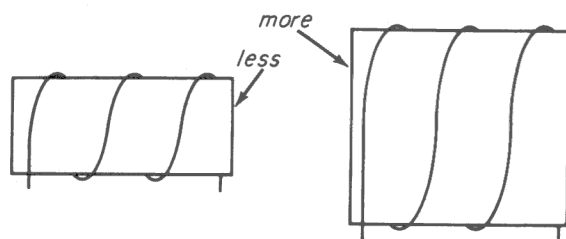
3. *The radius of the coil.* If two coils have the same number of turns and the same length but the radius of one is greater than that of the other, the coil with the greater



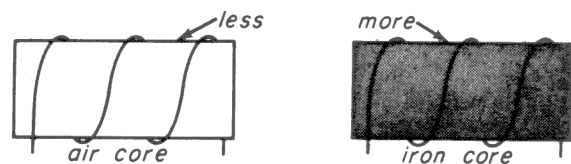
(a) inductance varies with number of turns



(b) inductance varies with length of coil



(c) inductance varies with coil radius



(d) inductance varies with permeability of coil

Fig. 14-6

radius has the greater inductance. (Fig. 14-6c).

4. *The permeability of the core.* If two coils have different cores but are alike in all other factors, the coil whose core has the greater permeability has the greater inductance (Fig. 14-6d).

To find the inductance of a single layer air-core coil, we can use the following formula:

$$L \text{ (in } \mu\text{h)} = \frac{r^2 N^2}{9r + 10l}$$

where:

N^2 is the total number of turns multiplied by itself

r is the radius of the coil in inches (half the diameter)

r^2 is the radius of the coil multiplied by itself

l is the length of the coil winding in inches

For example, an 80-turn coil has a winding length of two inches and is wound on a coil form one inch in diameter (the radius is 1/2 inch). The inductance of this coil would be:

$$\begin{aligned} L &= \frac{1/2 \times 1/2 \times 80 \times 80}{(9 \times 1/2) + (10 \times 2)} \\ &= \frac{1/4 \times 6,400}{4.5 + 20} \\ &= \frac{1,600}{24.5} \\ &= 65.3 \mu\text{h} \end{aligned}$$

14.3. INDUCTION IN A-C CIRCUITS

We have now seen the effects of self-inductance in a d-c circuit. Now let us see what effect self-inductance has in a circuit in which a sine wave alternating current is flowing. We know that the cemf is induced in a coil only when the current flow through the coil changes. This is because the magnetic flux must expand or contract and cut the turns of the coil to induce a cemf. The speed with which the current is changing, or its *rate of change*, determines the rate at which the magnetic flux cuts the conductor. The faster the current changes, the faster the magnetic flux cuts the conductor, and the larger the cemf becomes. If the current changed at a slower rate, the cemf would be smaller.

Rate of Change in a Sine Wave. In Theory Lesson 13, you learned how a sine wave representing current or voltage varies during one armature rotation of an a-c generator on

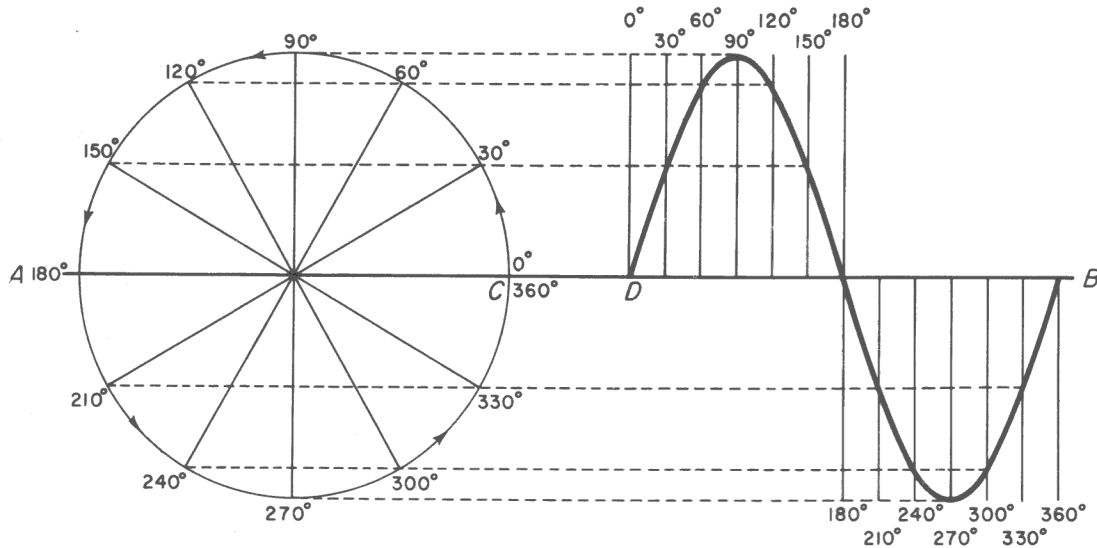


Fig. 14-7

one cycle. Fig. 14-7 shows how the current rises from a value of zero, at zero degrees of rotation, to its maximum value at the 90-degree rotation point. Notice, too, that these changes in current are shown for so many degrees of rotation instead of units of time. However, we can connect them together by knowing that each 90 degrees of rotation represents 1/4 of a cycle. Figure 14-8, shows the rate of change in a sine wave for

each 18-degrees of rotation from zero degrees to 90 degrees. During the first 18 degrees of rotation, the current changes from zero to 31 percent of its maximum value. In the next 18 degrees it reaches 59 percent of its maximum value, which represents a change of 28 percent ($59 - 31 = 28$). In the next 18 degrees of rotation it rises to 81 percent of maximum, which represents a change of 22 percent. The next 18-degree step shows a change of 14 percent, and the following step, only 5 percent. Let's look at these changes, one right after the other:

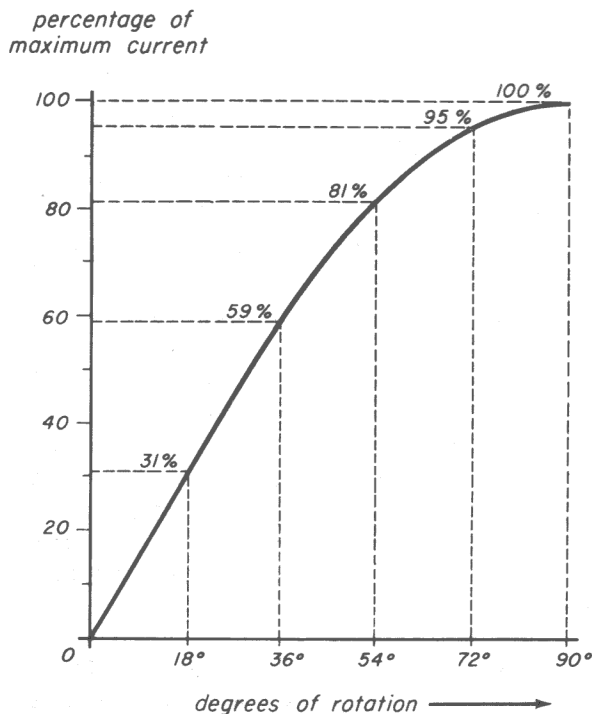


Fig. 14-8

0 to 18 degrees,	31 percent
18 to 36 degrees,	28 percent
36 to 54 degrees,	22 percent
54 to 72 degrees,	14 percent
72 to 90 degrees,	5 percent

You can see at a glance that the greatest rate of change takes place near zero degrees and that as the rotation nears the 90 degree point, the change becomes smaller and smaller. In a similar fashion the rate of change is maximum as the curve crosses the axis at the 180 degree point and at the 360-degree point.

Now that we have discussed the rate of change in sine wave alternating current, let us see the effect of this current in an in-

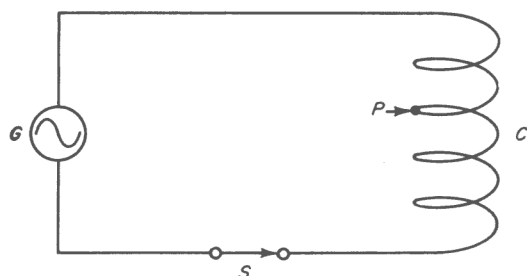


Fig. 14-9

ductive circuit. Figure 14-9 shows an a-c generator G connected to a coil C through a switch S . When the switch is closed, the current which flows will have the shape shown in Fig. 14-10a. The sine wave of current is divided into 90-degree segments (each quarter cycle) to help you follow the operation of the circuit. The first quarter cycle or 90-degrees is from A to B , the second from B to C . This represents the first half cycle because the current flow reverses direction at C and flows in the opposite direction for the last two quarter cycles from C to D and from D to E . If we could cut the coil at the point marked P in Fig. 14-9 without interrupting the flow of current, the cross section would look like Fig. 14-10b during the two half-cycles. Let us say that during the first half-cycle when the current wave is in the positive direction (above the axis) it is flowing out of the cross section, and during the negative half cycle it is flowing into the cross section. During the first half-cycle, the direction of the magnetic flux is clockwise with the direction of the flux reversing with the reversal of current in the second half cycle. Figure 14-10c shows a section of the wire in the coil which we will use to determine the direction of the induced voltage. Point M on this cross section will be used; however, the action at all other points around the cross section of the wire is the same.

Let us trace the action when an alternating current flows through a coil beginning at point A on Fig. 14-10a. Since the current is just passing through its zero value and increasing, the magnetic flux is just beginning to cut the wire at point M in Fig. 14-10c as it expands out from the wire. Since the flux direction is clockwise at point M the flux direction at this point is

upward. The conductor at point M appears to move to the right with respect to the magnetic flux. Now let us apply the three-finger rule, to determine the polarity of the induced voltage or *cemf*. Applying the left hand rule for induced voltage, we find that the thumb points to the right, the forefinger points up, and the middle finger points into the cross section, as shown by the arrow for induced *cemf* in Fig. 14-10c. This means that a voltage has been induced in the coil with a polarity that would tend to make current flow in a direction that is opposite to the original current. This is the same thing that happened when we were studying the d-c circuit. But now, just as the current passes the zero axis, the line is increasing at its most rapid rate, as you know from your study of the rate of change in a sine wave. Therefore, the *cemf* in Fig. 14-10a is maximum at point A and opposite in polarity to the current flow. When we talk about current flow or current we always mean the generator current as shown in Fig. 14-10a. This is the only current that actually flows through the coil.

As the current of Fig. 14-10a goes from point A to point B , its direction remains the same as it was at point A . The magnetic flux stays in the same direction and continues to move across point M from right to left in Fig. 14-10c. If we were to apply the left-hand rule at any time during this interval, we could see that the direction of the induced voltage is still the same as it was at point A . But, although the current is getting greater from point A to point B , its rate of change is getting smaller. The induced voltage, therefore, is also getting smaller and smaller as we go from point A to point B .

At the instant that the current reaches point B , it has its greatest value. The magnetic flux is still in the same direction as before, but, at this exact instant, the *rate of change* of current is zero, since the current is almost the same an instant before and an instant later. This means that the magnetic flux has stopped changing, so there is no motion between the wire and the magnetic flux. Therefore, the induced vol-

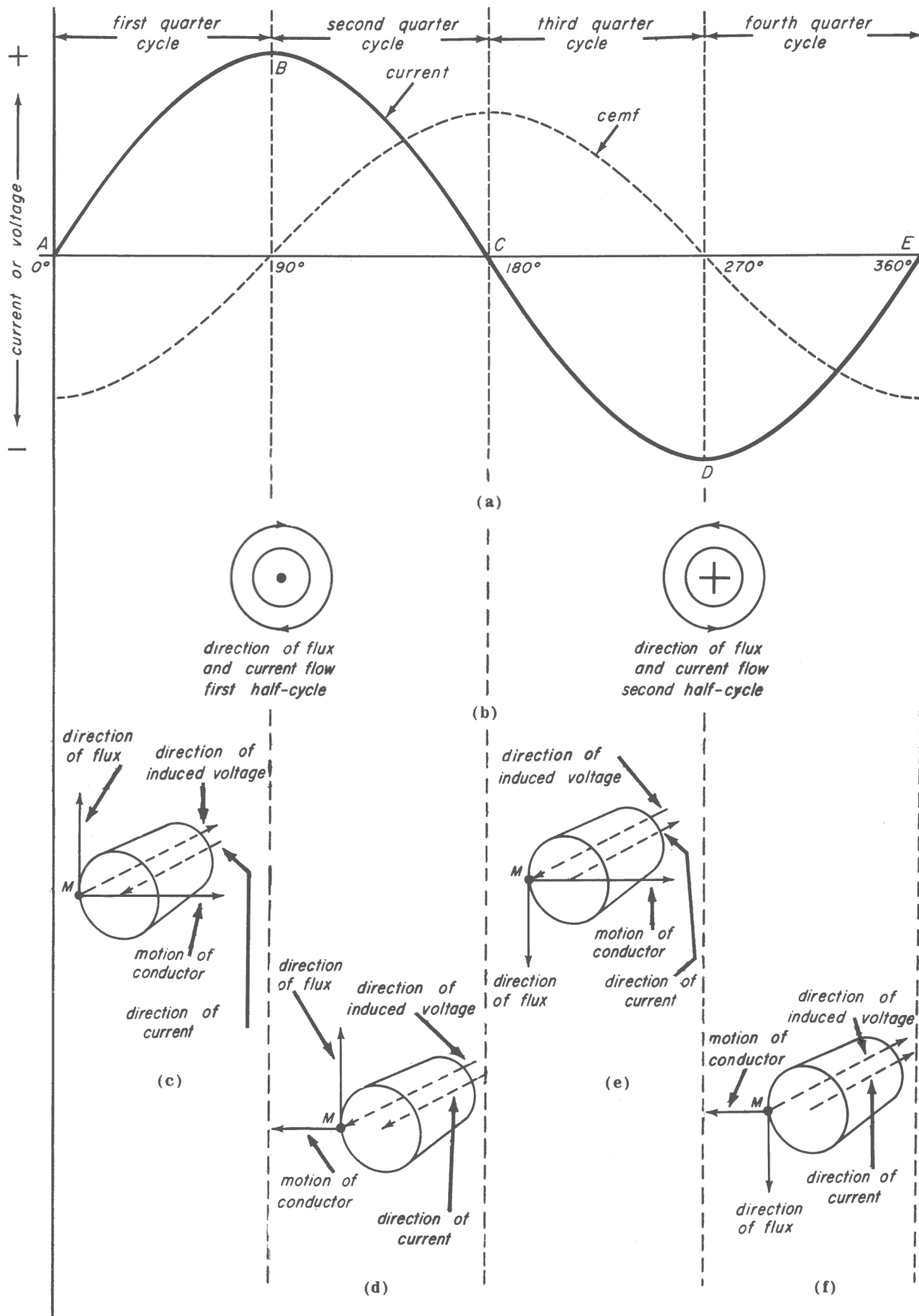


Fig. 14-10

tage becomes zero at this instant, even though the current is maximum.

Now the current starts to go from point *B* to point *C*. It is still flowing in the same direction as it was from point *A* to point *B*, so the magnetic flux is still in the same direction, as shown in Fig. 14-10*d*. But since the current is now getting smaller, the magnetic field is now collapsing around the wire. The flux moves back toward the center of the cross section or from left to right at point *M*. We can now say that point *M* on the wire appears to be moving from right to left with respect to the magnetic flux. If we apply the left-hand rule during this interval, we can see that the induced voltage now has the same polarity as the current, or would tend to make current flow in the same direction. Even though the polarity is the same, the *cemf* is increasing while the line current is decreasing. This is what we might expect because this is another example of Lenz's law operating. The current is trying to decrease to zero, but the induced voltage opposes this because its polarity tries to keep current flowing. Also, though the current is getting smaller between points *B* and *C* its rate of change is greatest as it passes through zero at point *C*. Therefore, the *cemf* in this interval increases to a maximum at point *C* with a polarity opposite to the polarity at point *A*.

As the current passes point *C*, two important things happen:

1. The current changes direction and flows into the cross section as shown by the plus sign in Fig. 14-10*b* for the second half cycle. This means, that the direction of magnetic flux also changes to counterclockwise. Looking at the cross section of the wire between points *C* and *D* the direction of the magnetic flux is down at point *M* as shown in Fig. 14-10*e*. The current arrow points into the cross section for the second half cycle and as the amount of current increases the magnetic flux again expands cutting the conductor at point *M* from right to left, so the conductor appears to be moving from left to right with respect to the flux. If we again apply the left-hand rule to Fig. 14-10*e*, the thumb points to the right, the forefinger points

down, and the middle finger points out of the center of the cross section.

2. The induced voltage is opposite in polarity to the current, but is still in the same direction as it was in Fig. 14-10*d* since the direction of the *cemf* has not changed. This is also shown in Fig. 14-10*a* where the *cemf* has the same direction for the second and third quarter cycles although the current reverses direction. The rate of change of current is getting smaller as we go from point *C* to point *D*, the induced voltage is decreasing from the maximum value it had at point *C*.

This action continues until we reach point *D*. At this point, although the current is again maximum in value, its rate of change is again zero. Therefore, the induced voltage becomes zero at this point. So, in going from *C* to point *D*, the induced voltage has the same direction as it had from point *B* to point *C*, but is decreasing from its maximum value at point *C* to zero at point *D*.

Now we come to the interval between point *D* and point *E*, shown in Fig. 14-10*f*. During this interval, the current is still in the same direction as in the interval before, but it is now decreasing from a maximum at point *D* to zero at point *E*. The magnetic flux is therefore in the same direction as before, but is collapsing. As we showed before, this makes the conductor at point *M* seem to be moving to the left with respect to the flux. Applying the left-hand rule, we can see that the induced voltage during this interval is again in the same direction, or has the same polarity as the line current. But even though the direction is the same, remember that the line current is trying to get smaller, so again the induced voltage opposes the current which produced it. Furthermore, the rate of change of current during this interval is increasing from zero at point *D* to maximum at point *E*. Therefore, the induced voltage during the interval from point *D* to point *E* has the same direction that it had during the interval from point *A* to point *B*, but it is increasing from zero at point *D* to maximum at point *E*.

We have now traced the current through the coil, and the voltage induced in the

coil because of this changing current, through one complete cycle. We have seen that the polarity of the induced voltage was such as to always oppose the change of circuit current. The induced voltage never causes current to flow, but only opposes the change in current already flowing through the coil. This is why the three-finger rule which you just used only pointed to the direction of the induced voltage.

Let us now examine the two sine waves shown in Fig. 14-10a. Though they both have the same shape or waveform, the graph of current crosses the zero voltage axis at the point marked zero degrees, and the graph of induced voltage crosses the zero-voltage axis 90 degrees later. Therefore, we can say that the current is leading the induced voltage by 90-degrees. Looking at the two waves, we can see that the shape of the graph of current from A to B in the first quarter cycle is the same as the shape of the induced voltage in the second quarter cycle. Similarly, the current in the second quarter cycle has the same shape as the induced voltage in the third quarter cycle. The current is *leading* the induced voltage by 90-degrees, or the induced voltage is *lagging* the current by 90-degrees. Both statements are the same.

Phase Difference. In speaking of voltages or currents which lead or lag each other, we use the term *phase*. Strictly speaking, phase is the difference in time between any point on a cycle and the beginning of that cycle. However, it is more important to express the difference of lead or lag between two voltages, two currents, or a voltage and a current. *Phase difference*, usually expressed in degrees, is the difference between the time when one wave passes the zero axis (at zero volts) and the time when a second wave passes the zero axis, both waves going in a positive direction. In Fig. 14-10a the cemf passes through the zero axis 90 degrees after the current. We could say that the cemf lags the current by 1/4 cycle, but it is usually more convenient to give the amount of lead or lag in degrees.

If two currents or voltages, or a current and a voltage rise and fall and change direc-

tion at the same rate and the same time, we say that they are *in phase*. When voltages or currents are not in phase — when there is even the smallest time difference in the rise and fall of one in relation to the other — we say that they are *out of phase*. In such cases, we say that one leads or lags the other by a *phase angle* of so many degrees — whatever the difference is.

Up to this point, we have discussed only the current that flows through a coil and the cemf induced by this current. We have not mentioned the generator voltage, or *applied voltage*, as it is usually called. It is this applied voltage that causes the current flow through the coil. In Fig. 14-11, the graph of the applied voltage and the resulting current flow is shown. When an alternating voltage is applied to a pure inductance (no resistance), the current tries to rise, but is prevented from rising by the cemf. When the applied voltage reaches its maximum at 90 degrees, the current is at zero. When the applied voltage reaches 180 degrees, the current is at its maximum. The current *lags* the applied voltage by 90 degrees.

You can get a better understanding of the 90-degree phase angle between applied voltage and current in a coil by considering what happens when you use a lawn mower.

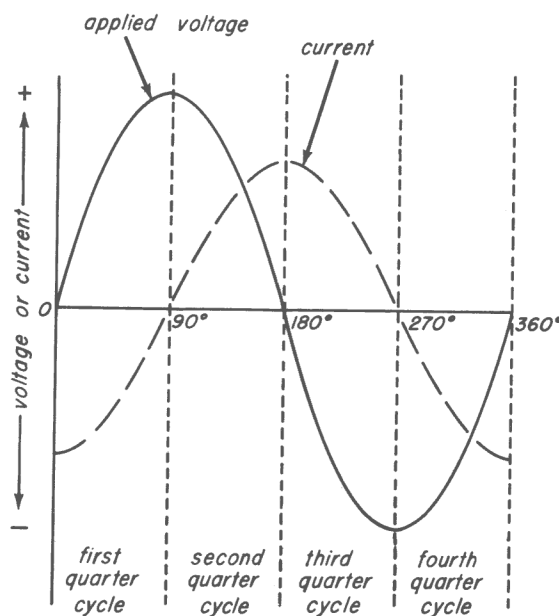


Fig. 14-11

The force needed to start the mower moving, or to keep it moving, is similar to the applied voltage across the coil. The speed with which the mower moves is similar to the current in the coil. When you begin to push the mower, a large force is needed to *start* it moving and it takes a little time to get the mower started. This is similar to what happens at the end of the first quarter cycle of Fig. 14-11, where the applied voltage is maximum, but the current is zero. As the mower begins to move, less and less force is needed to keep it moving, and yet the speed keeps increasing. This is similar to what happens during the second quarter cycle of Fig. 14-11, where the voltage is decreasing, but the current is increasing. Finally, when the mower reaches its maximum speed, a minimum of force is needed to keep it moving at this speed. Actually, if there were no friction between the mower and the ground, the force needed to keep it moving with maximum speed would be zero. This is similar to what happens at the end of the second quarter cycle of Fig. 14-11. If you want to slow the mower down just when it reaches its greatest speed, you have to pull the mower back, or apply a force in the opposite direction. As you pulled harder and harder, the mower would slow down more and more, but would still be moving. At the instant the mower stopped, you would be pulling with maximum force. This is similar to what happens during the third quarter cycle of Fig. 14-11. The voltage is increasing in size, but in the opposite (negative) direction. You should be able to use this explanation to see what force would have to be applied to the mower to move it back to its starting point. You can compare this with the last two quarter cycles of Fig. 14-11.

In Fig. 14-12 we examine the applied voltage, the current and the cemf on the same axis to determine the phase difference between all three values. We saw in Fig. 14-10a that the cemf lags the current by 90 degrees. Fig. 14-12 shows that the cemf is *equal* in size, and *opposite* in polarity to the applied voltage. Let us see why this is so. We learned that the cemf lags the current by 90 degrees, and that the current lags the applied voltage by 90 degrees. This means

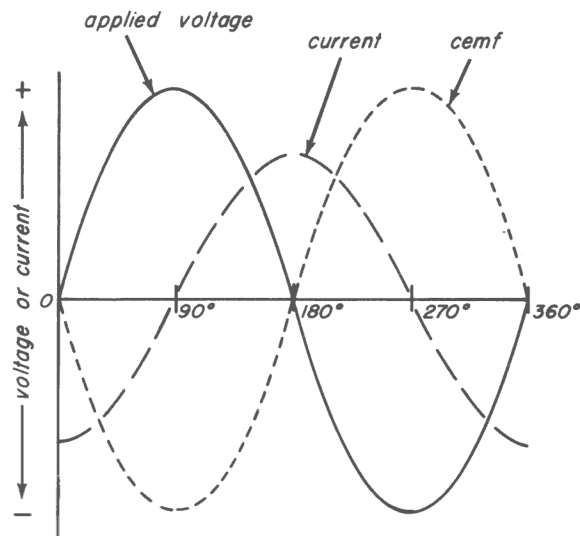


Fig. 14-12

that the cemf lags the applied voltage by 90 plus 90, or 180 degrees. Since 180 degrees is exactly 1/2 cycle, the cemf reaches its maximum negative when the applied voltage is at positive maximum. A phase difference of 180 degrees means that the voltages will oppose each other at all times.

14-4. INDUCTIVE REACTANCE

You have already seen how inductance opposes a change in current in a sine-wave a-c circuit. However, inductance has another important effect in an a-c circuit. Just as resistance offers opposition to setting up a current in a d-c circuit, inductance also offers opposition to setting up a current in an a-c circuit. The opposition that inductance offers a sine-wave current in an a-c circuit is called *inductive reactance*. It is represented by the symbol X_L , and is measured in ohms. Just as resistance was found to be equal to applied voltage divided by current, or E/I , inductive reactance is also equal to voltage across the coil divided by current through the coil. This can be expressed by the formula:

$$X_L = \frac{E}{I}$$

where:

X_L = inductive reactance of the coil in ohms

E = voltage across the coil in volts

I = current through the coil in amperes

This relationship is called Ohm's Law for a circuit containing pure inductance. In using this formula, we have to be careful about what values of voltage and current we use, since sine-wave voltage and current are always changing in value. We must, therefore, use some *particular* value for both voltage and current. We can use maximum voltage and current values, or effective voltage and current values. For example, if a sine-wave voltage reaches a maximum value of 100 volts, and the current reaches a maximum value of 10 amperes, the inductive reactance is:

$$X_L = \frac{E}{I}$$

$$= \frac{100}{10}$$

$$= 10 \text{ ohms}$$

$$\begin{array}{r} 70.7 \\ 7.07 \\ \hline 10 \end{array}$$

On the other hand, if we use the effective value of voltage, we must also use the effective value of current. If the effective value of a sine-wave voltage is 117 volts and the effective value of current is 11.7 amperes, then the inductive reactance is:

$$X_L = \frac{E}{I}$$

$$= \frac{117}{11.7}$$

$$= 10 \text{ ohms}$$

Just as in the case of resistance, Ohm's Law for a circuit containing pure inductance can be expressed in three ways:

$$X_L = \frac{E}{I}$$

$$E = I \times X_L$$

$$I = \frac{E}{X_L}$$

The first form is convenient for finding the inductive reactance when voltage and current are known, as we have already seen. The second can be used to find the voltage when current and inductive reactance are known, and the third for finding current when voltage and inductive reactance are known. For example, to find the voltage across a coil that has an inductive reactance of 25 ohms and in which a current of 5 amperes maximum value is flowing, we say,

$$E = I \times X_L$$

$$= 5 \times 25$$

$$= 125 \text{ volts (maximum)}$$

Remember that if maximum value of current is used, voltage must also be taken as maximum value, and vice versa. To find the current flowing through a coil that has an effective voltage of 150 volts across it and that has an inductive reactance of 75 ohms, we say,

$$I = \frac{E}{X_L}$$

$$= \frac{150}{75}$$

$$= 2 \text{ amps}$$

The inductive reactance of a coil also can be determined without using voltage and current values. It can be expressed by the formula:

$$X_L = 2\pi fL$$

where:

X_L = inductive reactance in ohms

π = a constant, approximately 3.14

f = frequency of the voltage and current wave, in cycles per second

L = inductance of the coil in henrys

For example, to find the inductive reactance of a coil with an inductance of 0.2 henry in a circuit in which the applied voltage has a frequency of 100 cycles per second, we say:

$$\begin{aligned} X_L &= 2\pi fL \\ &= 2 \times 3.14 \times 100 \times 0.2 \\ &= 125.6 \text{ ohms} \end{aligned}$$

At very high frequencies, the inductive reactance of a given coil becomes extremely great, and, therefore, offers very great opposition to current flow in an a-c circuit. As the frequency becomes smaller, the inductive reactance becomes smaller until, at zero frequency (d.c.), the inductive reactance is zero, no matter how great or little the inductance rating of the coil is. For this reason, we must be very careful about operating a coil at the frequency for which it was designed. For example, let us say that a 20-henry coil was designed to operate in a 110-volt, 60-cycle circuit. The inductive reactance of the coil at 60 cycles per second is

$$\begin{aligned} X_L &= 2\pi fL \\ &= 2 \times 3.14 \times 60 \times 20 \\ &= 7,536 \text{ ohms} \end{aligned}$$

When connected to a 110-volt, 60-cycle source, it would draw:

$$\begin{aligned} I &= \frac{E}{X_L} \\ &= \frac{110}{7,536} \end{aligned}$$

$$= 0.0146 \text{ amp or } 14.6 \text{ ma}$$

If the same coil were connected to a 110-volt 25-cycle circuit, the inductive reactance would be 3,140 ohms and the current would be more than doubled. This would cause the coil to overheat, which might cause the coil to become permanently damaged. If the same coil were connected in a 110-volt d-c circuit, it would draw a large enough current to burn out the coil.

You can better understand the effect of frequency on the inductive reactance of a coil if you apply voltage of different frequencies to the same coil. Table A shows the reactance of various coils as the frequency of the voltage applied changes.

TABLE A - HOW CHANGES IN FREQUENCY AND/OR INDUCTANCE AFFECT REACTANCE

Coil Inductance (henrys)	Reactance (ohms)						
	60 ~	120 ~	250 ~	500 ~	1,000 ~	10,000 ~	100,000 ~
0.01	3.77	7.55	15.7	31.4	62.8	628	6,280
0.1	37.7	75.5	157	314	628	6,280	62,800
0.5	188.5	377	786	1,570	3,140	31,400	314,000
1	377	755	1,570	3,140	6,280	62,800	628,000
10	3,770	7,550	15,700	31,400	62,800	628,000	6,280,000
50	18,850	37,700	78,600	157,000	314,000	3,140,000	31,400,000
100	37,700	75,500	157,000	314,000	628,000	6,280,000	62,800,000

You have learned two important effects of inductance in a sine-wave a-c circuit. You have seen that there is a phase difference of exactly 90 degrees between the applied voltage and the current, with the voltage leading the current, or the current lagging the voltage. You have also seen that the opposition to alternating current is in the form of inductive reactance.

14.5. VECTORS

An understanding of vectors is very useful in our study of radio. A vector may be simply defined as a quantity that has direction as well as magnitude. Let's break this down a little and see exactly what is meant. To begin with, let's take quantities with magnitude only. A pound of butter, a ton of coal, three yards of ribbon, ten miles, five hundred soldiers, all are quantities with magnitude but without direction. Even such a statement as, "I went for a 7-mile hike" discusses a quantity with magnitude only. This is true even though a 7-mile hike included one or more directions. However, if you say, "I hiked 7 miles from *O* in a northerly direction to *A*, as in Fig. 14-13, you have expressed a quantity with both magnitude and direction. What is more, if you have a map that shows the relative positions of *O* and *A*, you can draw a line between these two positions to represent the magnitude of your hike — as was done in the illustration. We call such a line a vector. So, we can see that there is still another definition for the term, vector. A vector is also a straight line with a starting point and a finishing point, that represents the magnitude and direction of a quantity.

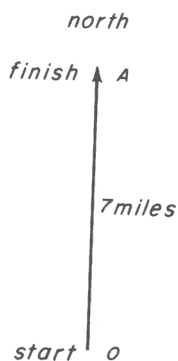


Fig. 14-13

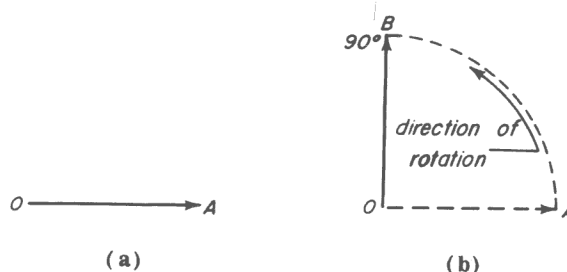


Fig. 14-14

When we speak of direction normally, we think of directions like north, east, south, and west, or in terms of up, down, left, and right. In our use of vectors, the directions we most often use are in terms of the number of degrees of rotation in a clockwise or counterclockwise direction. For example, Fig. 14-14a represents a vector of zero degrees rotation. In this vector, *O* represents the point of origin or the starting point of the vector and *A* represents the end of the vector. The vector has a direction from *O* to *A*, which is also shown by the arrow at the end of the vector. Figure 14-14b shows the same vector after a rotation of 90 degrees in a counterclockwise direction. The direction of this vector is from *O* to *B*.

In speaking of the phase relationship that exists between a current and a voltage or between two currents or two voltages, we use the phase angle — the angle that represents the difference in rotation between them. To be able to do this, we must have some standard starting point for a circle — we must know where zero degrees is. So we always make angular (having angles) rotations start from a horizontal position to the right of the origin *O*, which is the axis of rotation (if you were drawing a circle, point *O* would be the center). Figure 14-15a shows a vector of zero degrees. Rotation in a counterclockwise direction, as in Fig. 14-15b is considered *positive* and leading and, as in Fig. 14-15c, rotation in a clockwise direction is considered negative and lagging with respect to a zero-degree starting point. We show phase angle by this symbol, $\angle$. For example, a phase angle of zero degrees is written $\angle 0^\circ$, the angle in Fig. 14-15b is written $\angle 50^\circ$, and that in Fig. 14-15c is written $\angle -40^\circ$. When the

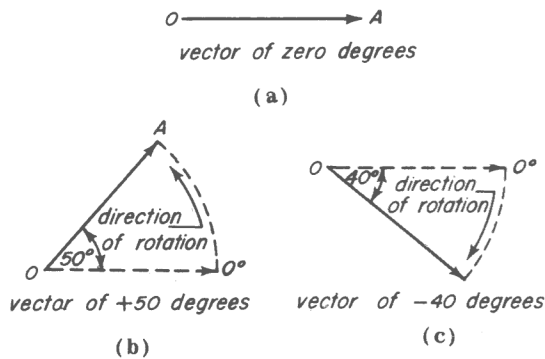


Fig. 14-15

angle is positive, the + sign is not necessary; but when the angle is negative, the - sign must be used.

Let's apply vectors to some typical currents and voltages and see how they are used. For example, in a circuit that contains only pure resistance, a-c current and voltage rise and fall at the same instant. This means that there is no phase difference between them. So, a resistive current or voltage is represented by a vector of zero degrees, as in Fig. 14-16. Here one vector represents a current of 10 amperes whose phase is zero (which we would write $I = 10/0^\circ$) and the other represents a voltage of 100 volts whose phase is also zero (which we would write $E = 100/0^\circ$). Notice that the voltage vector has been divided into ten parts, each of which represents 10 volts. For the sake of convenience, we did not use the same units to represent the size of current and voltage. If we did, the voltage vector would have been ten times as long as the current vector. If several voltages or currents are shown on the same diagram, all voltages must be drawn to the same scale, and all currents to the same current scale.

In Fig. 14-16, we drew the voltage and current vectors separately. Normally, we combine them in the same drawing, as in Fig. 14-17a. Because the voltage and current are in phase, they are shown on the

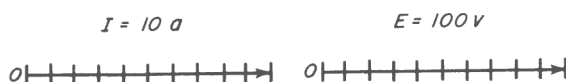


Fig. 14-16

same line. Notice that each vector has the same origin. When two or more voltages or currents are shown in phase, as in Fig. 14-17b, each current or voltage vector is drawn to the same scale. Notice that, in this case, each voltage does not have the same origin; instead, one vector adds to the next.

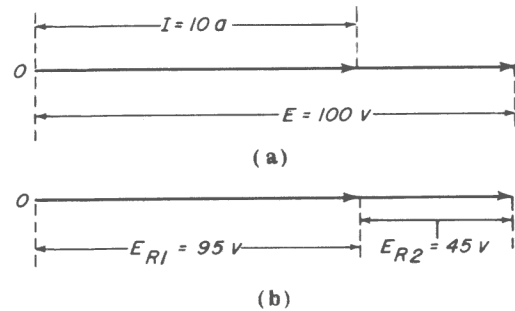


Fig. 14-17

Let's take the case of a voltage and a current that are not in phase with each other. You learned earlier in this lesson that voltage leads current by 90 degrees in a purely inductive a-c circuit, so our vector must show the voltage leading the current by 90 degrees. This is done by pointing the voltage vector upward, which really means rotating it 90 degrees counterclockwise from the current vector, as shown in Fig. 14-18. This vector drawing shows a voltage ($E = 100/90^\circ$) leading a current ($I = 10/0^\circ$) by 90 degrees. We can draw it as it is in Fig. 14-19 and still show the voltage leading the current by 90 degrees. The - sign shows an angle of lag.

14.6. POWER IN A-C INDUCTIVE CIRCUITS

We have already learned about power in d-c circuits containing resistance. There are

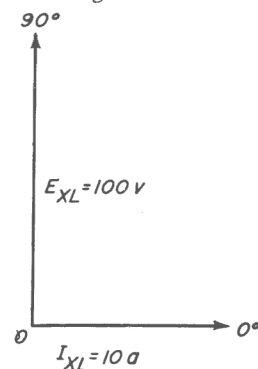


Fig. 14-18

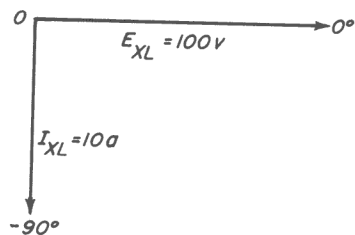


Fig. 14-19

three ways to express the power in such circuits. They are (1) $P = E \times I$, (2) $P = I^2 \times R$, and (3) $P = \frac{E^2}{R}$. The same formulas apply to a-c sine wave circuits, except that here P represents the *average* power, and I and E represent the *effective* or *r.m.s.* values of current and voltage respectively. Let us try to determine the average power in an a-c sine wave circuit containing only inductance. Remember that the current and voltage in an a-c circuit are changing from instant to instant. If the power at any instant is equal to the voltage times the current at that instant, we can draw the graph of power over a complete cycle. In Fig. 14-20, we have taken the graphs of voltage and current to get the graph of power at every instant in the cycle. Let us examine this power curve closely. You will see that wherever the voltage and current have the same sign, the power is *positive*, and wherever the voltage and current have opposite signs, the power is *negative*. Thus, in the first quarter-cycle, the voltage is positive (above the axis), and the current is negative (below the axis), so the power is negative. In the second quarter-cycle, both the current and voltage are positive, so the power is positive.

In the third quarter-cycle, the current is still positive but the voltage is negative, so the power is negative. Finally, in the last quarter-cycle, both the current and voltage are negative (same sign), so the power is again positive.

We can also see that wherever either voltage or current is zero, the power is also zero. Therefore, as shown in Fig. 14-20, the power is zero at the beginning of each interval. If we say that the positive power is power which the source (generator) is delivering to the coil, and negative power is power which the coil is returning to the source, we can begin to understand how

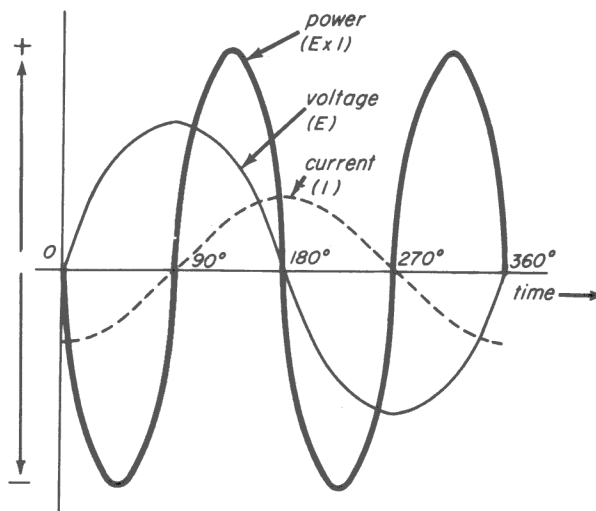


Fig. 14-20

much *average* power the source is delivering to the coil. During the intervals when the graph of power is positive the source delivers a certain amount of power to the coil. However, during the intervals when the graph of power is negative, the coil returns exactly the same amount of power to the source. Therefore, the average power delivered to the coil during a complete cycle is *zero*. This is always true in an a-c sine wave circuit containing *only* inductance (no resistance).

We can explain the fact that the average power in an a-c sine wave circuit containing only inductance is zero by considering the transfer of energy that takes place between the source and the coil during one complete cycle. During the second and fourth quarter-cycles, the coil is receiving power from the source, so it stores energy in its magnetic field by allowing the magnetic flux to build up from zero to a maximum. During the first and third quarter-cycles the coil returns power to the source. This means that the magnetic field collapses back to zero, and the coil releases the energy that was stored in the coil during the second and fourth quarter cycles.

Though the coil stores energy during the second and fourth quarter-cycles, the direction of the current is not the same during both intervals. Figure 14-20 shows that the direction of the current is reversed; if the direction of the magnetic field is in one

direction during the second quarter cycle, it is reversed during the fourth quarter cycle.

14-7. MUTUAL INDUCTANCE

You have now seen how a voltage is induced in a coil by a changing current flowing in the coil. The changing current set up a changing magnetic flux around the coil, so that the coil appeared to be cut by the changing flux. In Theory Lesson 12, you found that current flowing in one coil induced a voltage in another coil, which you were told was called transformer action. A better name for the process that causes a voltage to be induced in a coil because of a changing current flowing in another coil is *mutual induction*. It is similar to self induction, but there must be two coils in order to have mutual induction. The coils may be separate or they may be connected to each other, as they are in autotransformers.

You remember we were able to define the property of inductance for a coil in which a voltage was induced by self-induction. We can also define the property of *mutual inductance* for a pair of coils in which a voltage is induced by mutual induction. Mutual inductance is the property of a pair of coils that causes a voltage to be induced in *either* coil by a changing current flowing in in the *other* coil. Mutual inductance is represented by the letter M , and is also measured in henrys, millihenrys, or microhenrys. If we have two coils, L_1 and L_2 , in which currents I_1 and I_2 are flowing, we can express the mutual inductance M as follows:

$$M \text{ in henrys} = \frac{\text{voltage induced in coil } L_1}{\text{rate of change of } I_2}$$

$$= \frac{\text{voltage induced in coil } L_2}{\text{rate of change of } I_1}$$

In order for two coils to have mutual inductance, they must be properly *coupled*. This means that they must be fairly close to one another and must be fairly parallel to one another. The farther apart they are, or

the less parallel they are, the smaller the mutual inductance becomes. If they are very far apart, or if they are at right angles to each other, the mutual inductance is practically zero. We measure the amount of coupling between two coils by a number that is called *coefficient of coupling*. The coefficient of coupling between two coils can have values only between 0 and 1 and can be calculated as follows:

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

where

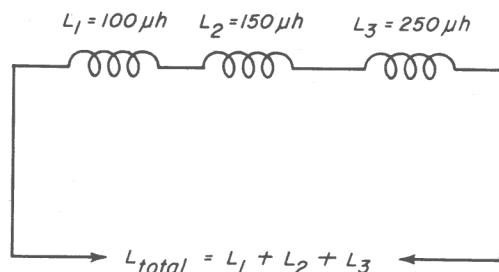
- K = the coefficient of coupling
 M = the mutual inductance in henrys
 L_1 and L_2 = respective inductances of the two coils in henrys.

(The radical sign $\sqrt{\quad}$ indicates that the square root of the quantity under the sign must be found. At the end of this lesson you will find two methods for finding squares and square roots. If a 1-henry coil and a 4-henry coil are coupled so that they have a mutual inductance of 1.5 henrys, the coefficient of coupling is:

$$\frac{1.5}{\sqrt{1 \times 4}} \quad \text{or} \quad \frac{1.5}{2} \quad \text{or} \quad 0.75$$

14-8. INDUCTORS IN SERIES AND PARALLEL

Just as we do with resistors, we sometimes have to work with inductors in series or parallel. If we have inductors in series, we also have to consider whether or not there is any mutual inductance between the coils. First, let us consider the case of several inductors connected in series, but without mutual inductance. If three inductors with inductances of 100 μh , 150 μh , and



*inductors in series with no
mutual inductance*

Fig. 14-21

250 μh are connected in series in such a way that there is no mutual inductance between them, as in Fig. 14-21, the total inductance is 100 + 150 + 250, or 500 μh . This can be expressed by the following formula:

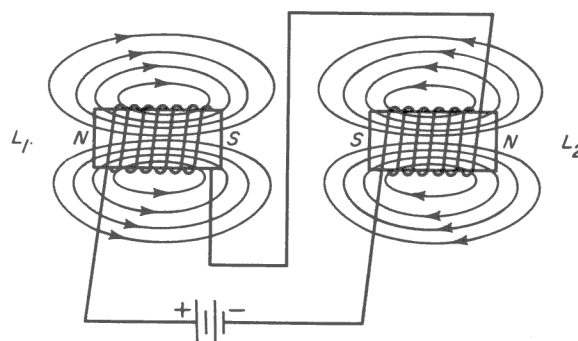
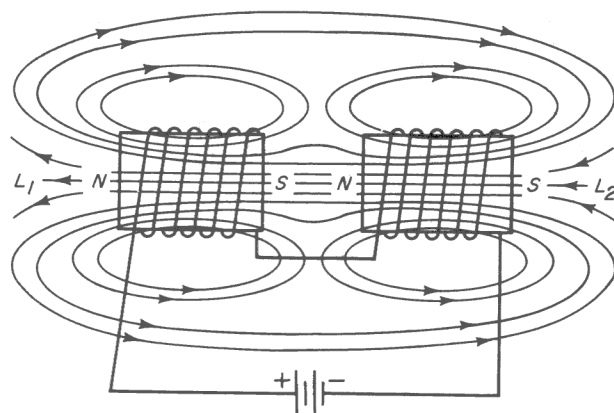
$$L_t = L_1 + L_2 + L_3 \dots \dots \text{etc.}$$

It is similar to the formula for finding the total resistance of several resistors in series. However, if two inductors are connected in series so that there is some mutual inductance between them, we must know two additional things besides the inductances of the two coils before we can calculate their total inductance. We must know the value of mutual inductance, and whether or not the coils are connected so their fluxes *aid* each other or *oppose* each other. The first case is called *series-aiding* and the second is called *series-opposing*.

Let's see what is meant by series aiding. Two coils are connected in series aiding when the flux of one induces a voltage in the other that adds to its cemf. Then, as in Fig. 14-22a, the total inductance is more than the simple addition of the two separate inductance values because of the amount contributed by the mutual inductance. The total inductance for coils connected in series aiding is then found by using the formula:

$$L_t = L_1 + L_2 + 2M$$

When two coils are connected so that there is mutual inductance, but the voltage induced in one by the other opposes (subtracts from) self-induced cemf, as shown in Fig. 14-22b,



$$L_t = L_1 + L_2 - 2M$$

Fig. 14-22

then the total inductance is less than the sum of the two separate inductance values. In fact, if the coefficient of coupling is 1 and the inductors, separately, have equal values of inductance, then their total inductance, when connected series opposing, will be zero. The total inductance for coils connected in series opposing is:

$$L_t = L_1 + L_2 - 2M$$

We generally do not consider mutual inductance when coils are connected in parallel. To find the total inductance of several inductances in parallel, we use the formula:

$$L_t = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \dots \dots \text{etc.}}$$

Note that this formula is similar to the one for finding the total resistance of several

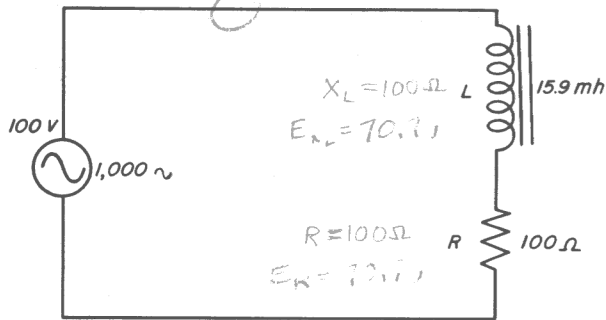


Fig. 14-23

resistors in parallel. For example, to find the total inductance of inductors of $100\ \mu\text{h}$, $200\ \mu\text{h}$, and $250\ \mu\text{h}$, in parallel, we proceed as follows:

$$\begin{aligned}
 L_t &= \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}} \\
 &= \frac{1}{\frac{1}{100} + \frac{1}{200} + \frac{1}{250}} \\
 &= \frac{1}{0.01 + 0.005 + 0.004} \\
 &= \frac{1}{0.019} \\
 &= 52.6\ \mu\text{h}
 \end{aligned}$$

14-9. CIRCUITS CONTAINING R AND L

Now that we have some knowledge of two circuit properties, resistance and inductance, let's see what happens when we combine them in one circuit. Suppose that we have the circuit shown in Fig. 14-23. It shows a 100-volt, 1,000-cycle a-c voltage source applied to a 15.9-mh coil in series with a 100-ohm resistor. We can find X_L , the inductive reactance of the coil, by using the formula:

$$\begin{aligned}
 X_L &= 2\pi fL \\
 &= 2 \times 3.14 \times 1000 \times 0.0159 \\
 &= 100\ \text{ohms (approx)}
 \end{aligned}$$

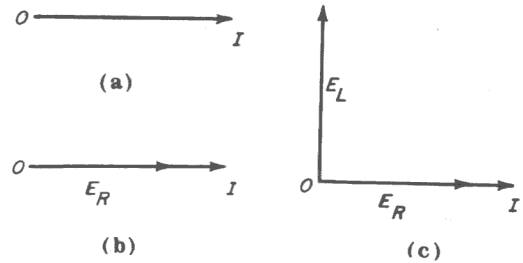


Fig. 14-24

Let's consider the current flowing in the circuit. We know that the current flowing in R must be in phase with the voltage across R , and the current flowing in L must lag the voltage across L by 90 degrees. But we also know, from our lessons about direct current, that *the current in all parts of a series circuit must be the same*. Therefore, the current flowing in R must be the same as the current flowing in L . Now let's see how the voltages across R and L are related to each other. If the circuit current is in phase with the voltage across R , then the voltage across R is in phase with the current. Also, if the circuit current lags the voltage across L by 90 degrees, then the voltage across L must lead the current by 90 degrees.

Let's see how these voltages and the current look on a vector diagram. Zero degrees is the normal reference point in vector diagrams. We'll use the current vector as our reference, as shown in Fig. 14-24a: The vector that represents the voltage across R (E_R) is in phase with the current, so it is drawn on the same line, as in Fig. 14-24b. The vector that represents the voltage across L (E_L) must lead E_R by 90 degrees and so must be drawn upward from the reference line, as in Fig. 14-24c. We know that $E_R = I \times R$ ($I \times 100$) and $E_L = I \times X_L$ ($I \times 100$), so each is equal to $100I$. From our lessons about direct current, we know that the sum of the voltage in a series circuit must equal the applied voltage. But we cannot add E_R to E_L and get $100I + 100I = 200I$, because the two voltages are not in phase. So, instead of adding them numerically, we add them vectorially. The easiest way to explain what we mean by vectorial addition is to show an example of how it is done. You'll find that

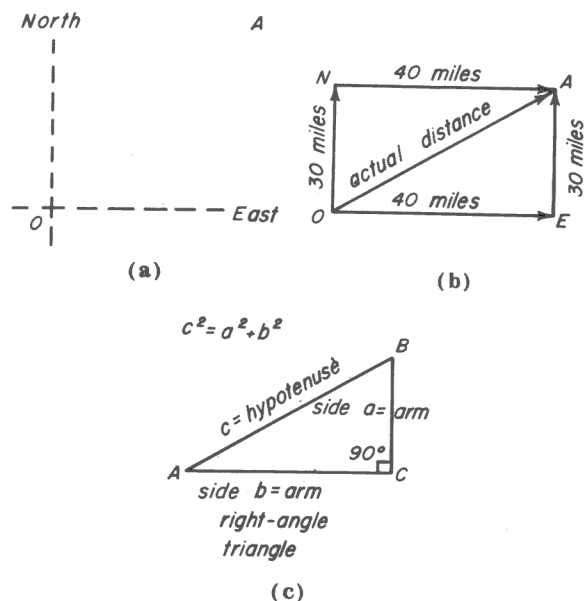


Fig. 14-25

it is not difficult at all, if you will follow closely the example given below.

Suppose that you are at point *O* in Fig. 14-25a and you want to go to point *A*. You look at a map and find that *A* is 30 miles farther north and 40 miles farther east than you are at point *O*. You could reach *A* by traveling 30 miles northward and then eastward for 40 miles. Or you could reach *A* by traveling 40 miles eastward and then 30 miles northward. No matter which of these two ways you used to travel to *A*, the distance you would actually be from *O* (when you reached *A*) is represented by the length of line from *O* to *A* (Fig. 14-25b). In Fig. 14-25c, a right angle triangle *ABC* is shown. Side *a* (opposite angle *A*) and side *b* (opposite angle *B*) are the two arms of the triangle, and side *c* (opposite the 90-degree angle) is the hypotenuse.

Also, the triangle formed between points *O*, *A*, and *E* in Fig. 14-25b is a right-angle triangle and the line *OA* is the hypotenuse. We know that *OE* = 40 and *EA* = 30, so:

$$(OA)^2 = (EO)^2 + (EA)^2$$

Take the square root of each side of the equation and get:

$$\begin{aligned} \sqrt{(OA)^2} &= \sqrt{(EO)^2 + (EA)^2} \\ &= \sqrt{40^2 + 30^2} \\ &= \sqrt{1,600 + 900} \\ &= \sqrt{2,500} \\ &= 50 \text{ miles} \end{aligned}$$

So, the actual distance between *O* and *A* is 50 miles.

Now let's apply this method to the voltage vector diagram. As shown in Fig. 14-26a, the vector diagram that you saw in Fig. 14-24c has been redrawn to scale. The current vector has been omitted, so all that you see are the two voltage vectors. Each vector is the same length as the other and is equal to 100V. To add these vectors vectorially, we draw a line from the arrow tip of the E_L vector (parallel to the E_R vector), and another line from the arrow tip of the E_R vector (parallel to the E_L vector) until it meets the line that we just drew. This line is exactly equal to the E_L vector and represents it in the voltage triangle in

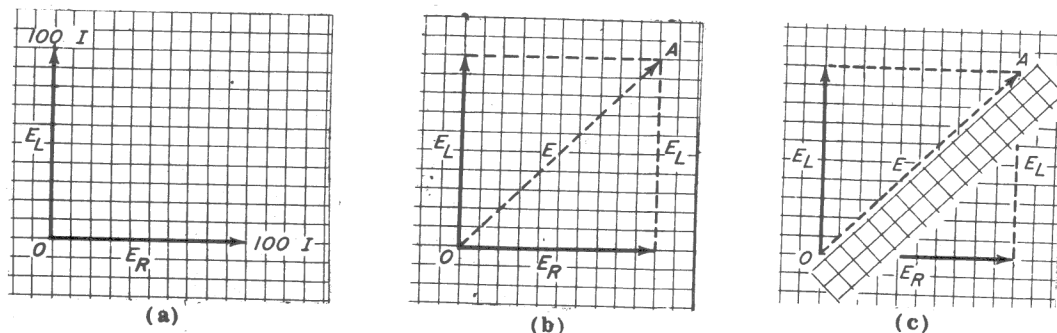


Fig. 14-26

Fig. 14-26b. Let's call the point where the two lines meet point *A*. A line drawn between *O* and *A* represents the vectorial sum of E_L and E_R , which equals the applied voltage, E . Because this vector diagram was drawn to scale (the scale used for E_R is the same as that used for E_L), we can measure vector *OA* and get an approximate idea of its value. Using the same scale as for E_R and E_L , as in Fig. 14-26c, we find that the applied voltage is about 1.4 times longer than either the reactive or resistive voltage ($100I$). Multiplying $100I$ by 1.4, we find that the applied voltage is equal to about 140 times I .

We can check this figure by using the Pythagorean theorem.

$$\begin{aligned} E &= \sqrt{E_R^2 + E_L^2} \\ &= \sqrt{100^2 + 100^2} \\ &= \sqrt{20,000} \\ &= 141 I \end{aligned}$$

This figure checks very closely with what we obtained by measuring with the scale in Fig. 14-26c.

If we knew how much the current was, we would have no trouble getting the answer. So the next step is to find out how much current is flowing in the circuit. To find this, we should know how much opposition there is to current flow. When an a-c circuit contains reactance and resistance, we call their combined opposition to current flow *impedance*, which we measure in ohms, abbreviated Z . We can now express Ohm's Law for any a-c circuit by the formulas

$$\begin{aligned} E &= I \times Z \\ I &= \frac{E}{Z} \\ Z &= \frac{E}{I} \end{aligned}$$

We can prove mathematically that $Z^2 = R^2 + X_L^2$. (If you are rusty in your algebra,

you can skip the rest of this paragraph and continue with the beginning of the next paragraph.) We have already learned that

$$E = \sqrt{E_R^2 + E_L^2}$$

If we replace E_R by IR and E_L by IX_L , we can write

$$E = \sqrt{I^2 R^2 + I^2 X_L^2}$$

We factor out I^2 and get:

$$\begin{aligned} E &= \sqrt{I^2 (R^2 + X_L^2)} \\ &= I \sqrt{R^2 + X_L^2} \end{aligned}$$

If we bring I to the left side of the equation, we get

$$\frac{E}{I} = \sqrt{R^2 + X_L^2}$$

But $\frac{E}{I} = Z$, so

$$\begin{aligned} Z &= \sqrt{R^2 + X_L^2} \\ Z^2 &= R^2 + X_L^2 \end{aligned}$$

which is what we set out to prove.

If we know the applied voltage and can calculate the value of the impedance (Z) of a circuit, we can then find the current flowing in the circuit. We know that the applied voltage is 100 volts (Fig. 14-23), X_L is 100 ohms, and R is 100 ohms. So we find:

$$\begin{aligned} Z &= \sqrt{R^2 + X_L^2} \\ &= \sqrt{100^2 + 100^2} \\ &= \sqrt{20,000} \\ &= 141 \text{ ohms} \end{aligned}$$

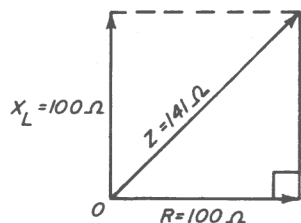


Fig. 14-27

This is shown in vector form in Fig. 14-27. The current may now be found as follows:

$$\begin{aligned}
 I &= \frac{E}{Z} \\
 &= \frac{100}{141} \\
 &= 0.707 \text{ ampere}
 \end{aligned}$$

You remember that E_R and E_{X_L} are each equal to $100I$, which is 100×0.707 or 70.7

volts. The applied voltage E_Z may be calculated as follows:

$$\begin{aligned}
 E_Z &= \sqrt{E_R^2 + E_{X_L}^2} \\
 &= \sqrt{70.7^2 + 70.7^2} \\
 &= \sqrt{10,000} \\
 &= 100 \text{ volts (as given)}
 \end{aligned}$$

We have just seen that $Z^2 = R^2 + X_L^2$. Figure 14-27 shows that the triangle formed by the Z , R , and X_L vectors is similar to the triangle formed by the voltage vectors. In fact, the phase angle is exactly the same for both. After you study capacitance (the one remaining circuit property, which is discussed in the next lesson), you will learn how to calculate the phase angle for circuits containing combinations of any circuit properties.